Scheme for Generating a Four-photon Entangled State for Quantum Information Processing

Yuuki Tokunaga†

Abstract

A scheme for generating a four-photon entangled state, which is an important resource for two-qubit quantum operations such as a controlled-NOT gate, is described. This scheme is constructed with four photons from parametric down-conversion, linear optical devices, and photon detectors, all of which are available using current technology. A scheme for demonstrating a controlled-NOT gate using the four-photon state as a resource is also described.

1. Introduction

Quantum information processing such as quantum computation or quantum cryptography is attracting a lot of attention. It is known that if we can perform arbitrary one-qubit (quantum bit) operations and a two-qubit interacting operation such as a controlled-NOT (CNOT) gate, then we can construct any quantum operation for quantum computation. One of the main difficulties in achieving quantum information processing is to construct the CNOT gate. For example, a nonlinear optical Kerr medium could be used for the CNOT gate, but the effect is too weak, so it is not practical as it is. Recently, it has become clear that quantum entanglement can be a resource of quantum information processing, for example, a resource for teleporting an unknown quantum state [1]. Gottesman and Chuang pointed out that a four-qubit entangled state

$$|\chi\rangle = \frac{1}{2}\left[\left(|00\rangle + |11\rangle\right)|00\rangle + \left(|01\rangle + |10\rangle\right)|11\rangle\right] \qquad (1)$$

is a resource for a CNOT gate [2]. They showed that a quantum teleportation of two qubits using $|\chi\rangle$ as a resource becomes a CNOT gate of the two qubits (**Fig. 1**). Furthermore, Raussendorf and Briegel [3] showed that special multipartite entangled states called "cluster states" can be used for a measurement-based quantum computation. The state $|\chi\rangle$ is also important in the sense that it is equivalent to a four-qubit cluster state

$$|C_4\rangle = \frac{1}{2}\left(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle\right) \qquad (2)$$

under a local unitary transformation.

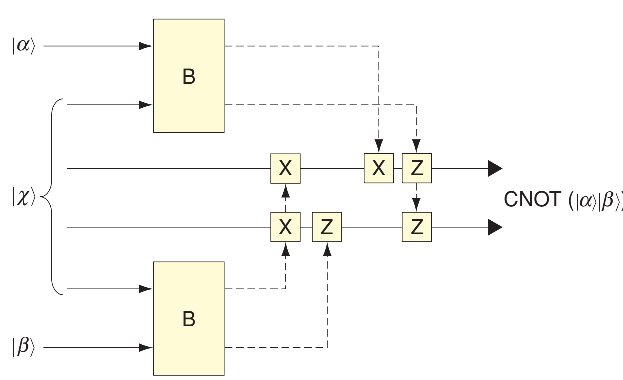

Fig. 1. A controlled-NOT gate achieved through quantum teleportation [2]. $|\alpha\rangle|\beta\rangle$ is the input to the CNOT gate. $|\chi\rangle$ is the required resource for the gate. B represents Bell measurement. Dotted lines are classical information flows. X and Z represent the operations corresponding to Pauli matrices σ_x and σ_z, respectively. These operations are done depending on the results of Bell measurements.

† NTT Information Sharing Platform Laboratories
Yokosuka-shi, 239-0847 Japan
Email: tokunaga.yuuki@lab.ntt.co.jp

In this paper, we report a simple experimental scheme [4] for preparing the four-photon entangled states $|C_4\rangle$ and $|\chi\rangle$ in Sec. 2. This scheme can be constructed from four photons produced by parametric down-conversion (PDC) [5], polarizing beamsplitters, half-wave plates, and conventional photon detectors. The successful events of preparing $|C_4\rangle$ are selected by coincidence detection. Our scheme does not require the optical paths to be stable to subwavelength precision. An experimental scheme [4] for a teleportation-based CNOT gate using $|\chi\rangle$ is also described in Sec. 3 to show the power of $|\chi\rangle$ as a resource.

2. Four-photon entangled state generation schemes

There are several methods of generating the four-photon entangled state $|C_4\rangle$. One scheme [6] requires the optical paths to be stable to subwavelength precision for interferometric stability. In another scheme [7], $|C_4\rangle$ is prepared by applying a probabilistic CNOT gate [8]-[10] between two entangled photon pairs obtained from PDC. It uses a polarization-dependent beamsplitter and the success probability is 1/9. If we use another probabilistic CNOT gate [11], the success probability increases to 1/8, and the success probability further increases to 1/4 if feed-forward control is applied [11], [12]. However, we need an ancillary photon to operate the gate; therefore, one needs five photons to prepare $|C_4\rangle$. A third scheme [13] requires one entangled photon pair and two single photons to obtain $|C_4\rangle$. However, when this scheme is used in a real experiment, if we substitute the two single photons with another pair of photons from PDC, then multiphoton generation on the same optical path causes errors, even if we allow for postselection.

Here, we describe a simple scheme [4] for generating $|C_4\rangle$ (**Fig. 2**) that has fewer requirements and/or a greater yield compared with the above-mentioned methods. This scheme requires only four photons from PDC. It starts with one entangled photon pair and adds two other single photons by applying beamsplitter operations. Unlike the scheme in Ref. [13], even if we substitute the two single photons with another pair of photons from PDC, multiphoton generation errors from PDC can be eliminated by postselection, as described later. Furthermore, this scheme does not require the optical paths to be stable to subwavelength precision. The successful events are postselected in the coincidence basis. The success probability is 1/4 provided that an entangled photon pair and two single photons are generated.

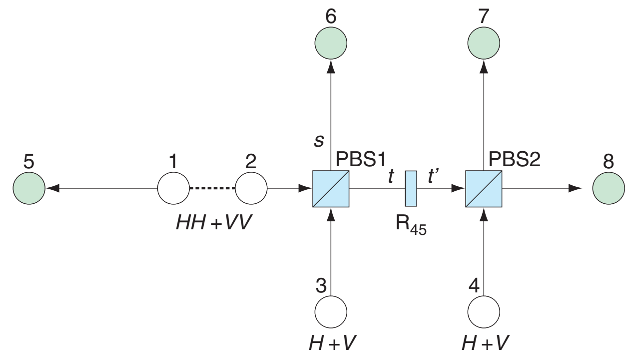

Fig. 2. Schematic diagram for generating a four-photon polarization entangled state $|C_4\rangle$.

The scheme is described in detail below. First, we assume that a polarization entangled photon pair is prepared in spatial modes 1 and 2, and that two single photons are also prepared in spatial modes 3 and 4 as follows:

$$\frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2\right)\otimes\frac{1}{\sqrt{2}}\left(|H\rangle_3+|V\rangle_3\right)\otimes\frac{1}{\sqrt{2}}\left(|H\rangle_4+|V\rangle_4\right). \tag{3}$$

Here, for example, $|H\rangle_1$ represents the state of a photon with horizontal polarization in spatial mode 1, and $|V\rangle_1$ represents the state of a photon with vertical polarization in spatial mode 1. Since a polarizing beam splitter (PBS) transmits horizontally polarized photons and reflects vertically polarized ones, the state of photons in modes 1, 2, and 3 transformed into

$$\frac{1}{2}\left(|H\rangle_1|H\rangle_s|H\rangle_t+|V\rangle_1|V\rangle_s|V\rangle_t\right) \tag{4}$$

after the photons have passed through PBS1 and we keep only the terms having one photon in each of the output modes 1, s, and t. The half-wave plates rotate the polarizations of the three photons in modes t by 45°, i.e.,

$$|H\rangle_t\rightarrow\left(|H\rangle_{t'}+|V\rangle_{t'}\right)/\sqrt{2},$$
$$|V\rangle_t\rightarrow\left(|H\rangle_{t'}-|V\rangle_{t'}\right)/\sqrt{2},$$

resulting in

$$\frac{1}{2}\left(|H\rangle_1|H\rangle_s|H\rangle_t + |V\rangle_1|V\rangle_s|V\rangle_t\right)$$
$$\rightarrow \frac{1}{2\sqrt{2}}\left(|H\rangle_1|H\rangle_s|H\rangle_{t'} + |H\rangle_1|H\rangle_s|V\rangle_{t'} + |V\rangle_1|V\rangle_s|H\rangle_{t'} - |V\rangle_1|V\rangle_s|V\rangle_{t'}\right). \quad (5)$$

After the photons in modes t' and 4 have passed through PBS2, we obtain the four-photon entangled state

$$\frac{1}{4}\left[|H\rangle_5|H\rangle_6|H\rangle_7|H\rangle_8 + |H\rangle_5|H\rangle_6|V\rangle_7|V\rangle_8 + |V\rangle_5|V\rangle_6|H\rangle_7|H\rangle_8 - |V\rangle_5|V\rangle_6|V\rangle_7|V\rangle_8\right] \quad (6)$$

by keeping only the terms having one photon in each of the output modes 5, 6, 7, and 8. This state is equivalent to state (2). The successful events of obtaining $|C_4\rangle$ can be postselected by four-photon coincidence detection. The success probability is 1/4 on the condition that an entangled photon pair in modes 1 and 2 and two single photons in modes 3 and 4 are provided. The state $|\chi\rangle$ is also obtained by rotating the polarizations of the photons in modes 1 and s by 45°.

Next, we consider the case in which we use PDC to generate the photons for input modes 1, 2, 3, and 4. In PDC, the photon pair generation rate per pulse γ is ~ 10^{-4} in typical experiments. Successful events of preparing $|C_4\rangle$ are obtained only when an entangled photon pair is generated in modes 1 and 2 by PDC, and two single photons are generated in modes 3 and 4 by another PDC. Such events occur with a rate of $O(\gamma^2)$. On the other hand, with a rate of this order, two-photon pairs are generated in modes 1 and 2 (or in modes 3 and 4) with modes 3 and 4 (1 and 2) being left in the vacuum. This contribution could lead to errors, but we can eliminate these failure events by postselection as follows. If two-photon pairs are produced in modes 1 and 2, then three photons never exist in output modes 6, 7, and 8. If two-photon pairs are produced in modes 3 and 4, then no photon will exist in output mode 5. The errors only occur when three (or more) photon pairs are produced by PDC with a small rate of $O(\gamma^3)$. The dark counting rate of conventional photon detectors is also quite low for fourfold coincidence detection.

3. Teleportation-based CNOT gate using $|\chi\rangle$

As described in [13], [14], a teleportation-based CNOT gate using $|\chi\rangle$ can be implemented by adding two photons as an input and by applying probabilistic Bell measurements using beamsplitting operations. In this case, we need six photons for the demonstration, and the success probability is 1/4 on the condition that $|\chi\rangle$ and the input state of two photons are provided. There is no need for the optical paths to be stable to subwavelength precision.

Here, we describe another experimental scheme [4] for a teleportation-based CNOT gate (**Fig. 3**), which uses only four photons and the teleportation deterministically succeeds in principle on the condition that $|\chi\rangle$ is given, though the input state of the CNOT gate is limited to a known product state (or a separable mixed state in general) and the scheme requires the optical paths to be stable to subwavelength precision. The basic idea of the teleportation in this scheme relies on previous schemes [15], [16], which we modified such that the teleportation can be simply applied after $|\chi\rangle$ has been given. This scheme can be demonstrated using currently available technology in

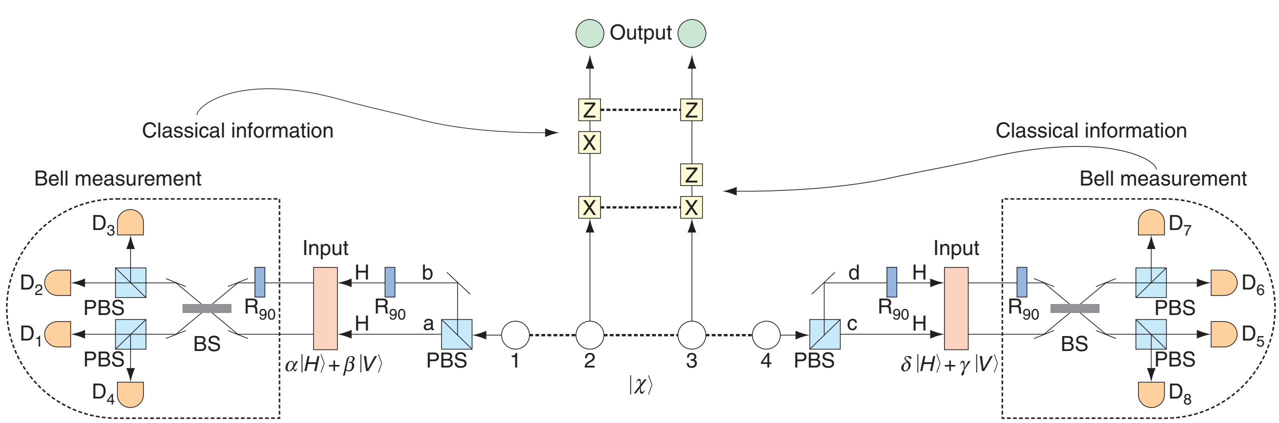

Fig. 3. Schematic diagram of a teleportation-based controlled-NOT gate. The parts surrounded by dotted lines correspond to Bell measurements.

combination with the scheme for preparing $|\chi\rangle$ presented in Sec. 2. Since the preparation scheme in Sec. 2 probabilistically succeeds on the coincidence basis, the successful events of the CNOT gate should be finally selected by coincidence detection.

Details of our scheme are given below. We assume that $|\chi\rangle$ is provided in polarization qubits of photons 1, 2, 3, and 4 as follows:

$$|\chi\rangle = \frac{1}{2}\Big[\big(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2\big)|H\rangle_3|H\rangle_4 + \big(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2\big)|V\rangle_3|V\rangle_4\Big]. \quad (7)$$

Photons 1 and 4 pass through polarizing beamsplitters, after which we obtain

$$\frac{1}{2}\Big[\big(|a\rangle_1|H\rangle_1|H\rangle_2 + |b\rangle_1|V\rangle_1|V\rangle_2\big)|H\rangle_3|c\rangle_4|H\rangle_4 + \big(|a\rangle_1|H\rangle_1|V\rangle_2 + |b\rangle_1|V\rangle_1|H\rangle_2\big)|V\rangle_3|d\rangle_4|V\rangle_4\Big], \quad (8)$$

where $|a\rangle_1|H\rangle_1$ $\big(|c\rangle_4|H\rangle_4\big)$ represents the state of a photon with horizontal polarization in path *a* (*c*), and $|b\rangle_1|V\rangle_1$ $\big(|d\rangle_4|V\rangle_4\big)$ represents the state of a photon with vertical polarization in path *b* (*d*). Labels 1 and 4 now indicate double channels that lead to Bell measurements. The half-wave plates in paths *b* and *d* (labeled as R_{90} in Fig. 3) rotate the polarizations of the photons by 90°. We then obtain

$$\frac{1}{2}\Big[\big(|a\rangle_1|H\rangle_2 + |b\rangle_1|V\rangle_2\big)|H\rangle_3|c\rangle_4 + \big(|a\rangle_1|V\rangle_2 + |b\rangle_1|H\rangle_2\big)|V\rangle_3|d\rangle_4\Big]|H\rangle_1|H\rangle_4. \quad (9)$$

If we regard (i) $|a\rangle$ and $|b\rangle$ as $|0\rangle$ and $|1\rangle$ for photon 1, (ii) $|c\rangle$ and $|d\rangle$ as $|0\rangle$ and $|1\rangle$ for photon 4, and (iii) $|H\rangle$ and $|V\rangle$ as $|0\rangle$ and $|1\rangle$ for photons 2 and 3, then we obtain an entangled state that is equivalent to Eq. (1) and the polarizations of photons 1 and 4 are free from this entanglement, so we can transform the polarizations of photons 1 and 4 into arbitrary product states

$$\frac{1}{2}\Big[\big(|a\rangle_1|H\rangle_2 + |b\rangle_1|V\rangle_2\big)|H\rangle_3|c\rangle_4 + \big(|a\rangle_1|V\rangle_2 + |b\rangle_1|H\rangle_2\big)|V\rangle_3|d\rangle_4\Big] \big(\alpha|H\rangle_1 + \beta|V\rangle_1\big)\big(\gamma|H\rangle_4 + \delta|V\rangle_4\big). \quad (10)$$

Here, the state $\big(\alpha|H\rangle_1 + \beta|V\rangle_1\big)\big(\gamma|H\rangle_4 + \delta|V\rangle_4\big)$ is assumed to be the input to the CNOT gate. After that, photon 1 (4) passes through the optical elements and should be detected by one of the detectors D_1, D_2, D_3, and D_4 (D_5, D_6, D_7, and D_8). Here, the optical elements and the detectors surrounded by the dotted lines in Fig. 3 correspond to Bell measurements (see [16]). Applying a unitary transformation $I \otimes I$, $\sigma_x \otimes \sigma_x$, $I \otimes \sigma_z$, or $\sigma_x \otimes \sigma_z \sigma_x$ to photons 2 and 3 according to detection at D_5, D_6, D_7, or D_8, and applying a unitary transformation $I \otimes I$, $\sigma_x \otimes I$, $\sigma_z \otimes \sigma_z$, or $\sigma_z \sigma_x \otimes \sigma_z$ to photons 2 and 3 according to detection at D_1, D_2, D_3, or D_4, we always obtain the same output state

$$\alpha\gamma|H\rangle_2|H\rangle_3 + \alpha\delta|V\rangle_2|V\rangle_3 + \beta\gamma|V\rangle_2|H\rangle_3 + \beta\delta|H\rangle_2|V\rangle_3. \quad (11)$$

This is exactly the state that is obtained by applying the CNOT gate to the target qubit in state $\alpha|H\rangle_1 + \beta|V\rangle_1$ and the control qubit in $\gamma|H\rangle_4 + \delta|V\rangle_4$.

4. Conclusion

Our simple experimental scheme for generating the four-photon entangled state $|C_4\rangle$ and $|\chi\rangle$ can be achieved with four photons produced by PDC, linear optical devices, and conventional photon detectors. It does not require the optical paths to be stable to sub-wavelength precision. The successful events are selected by coincidence detection, and the main errors from multiphoton PDC emissions are also eliminated by postselection.

When we use such a generation scheme, we should verify whether or not the generated state is a desired one. We have also proposed verification methods for efficiently verifying the state with high precision [17]. The generated entangled state is useful for investigating the entanglement of cluster states [17]-[20] or demonstrating the basic quantum gates of certain quantum computational models [2], [3], [21], [22]. The experimental scheme for the CNOT gate in Sec. 3 can be thought of as a simple scheme for demonstrating deterministic linear optical quantum computation [22] in the sense that (9) is the resource (called a "linked state") of a quantum circuit that has only one CNOT gate in this model. The probabilistic preparation of resource states in the coincidence basis is not scalable as it is, but it could become scalable for quantum computation if postselection is done without losing photons. This could be achieved by using quantum nondemolition measurement techniques, e.g., [23], [24], and by combining it with a near-deterministic teleportation scheme [21] or measurement-based optical quantum computation approach using cluster states [25], [26].

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Yuuki Tokunaga
Information Security Project, NTT Information Sharing Platform Laboratories.

He received the bachelor degree of integrated human studies in mathematical science from Kyoto University, Kyoto, and the M.S. degree in information science from the University of Tokyo, Tokyo, in 1999 and 2001, respectively. He joined NTT Information Sharing Platform Laboratories in 2001. Since then, he has been engaged in research on quantum information science, especially on algorithms, cryptography, and optics. Recently, he has been engaged in research on optical quantum information processing. He is currently studying for a doctor's degree through the University of Osaka. He is a member of the Physical Society of Japan.