

Electron Spin Rotation and Quantitative Determination of Spin-Orbit Coefficients

Takaaki Koga and Yoshiaki Sekine

Abstract

In recent research, there have been proposals to realize quantum bits (qubits) with electron spins, where these qubits compose a quantum computer. In this article, we explain the mechanism of electron spin precession that is caused by the Rashba spin-orbit interaction and introduce the latest research results on quantitative determination of the Rashba and Dresselhaus spin-orbit coefficients. These findings represent a step toward the utilization of electron spins as qubits of a quantum computer.

1. Background

1.1 Spin in conventional electronic devices

An electron has not only a *charge* degree of freedom but also a *spin* degree of freedom associated with the spin angular momentum. If a charged particle has angular momentum, magnetic moment is generated. Therefore, each electron, having a spin angular momentum, acts as a micromagnet. The spin degree of freedom can be interpreted, in one way, as the freedom in the orientation of the micromagnet. In other words, spin is not a conserved quantity. Thus, it has not been easy to use them in conventional devices. Recently, research on electron spin in semiconductors has become increasingly important because future technologies that will transcend existing concepts, such as quantum computers based on spin qubits, will be based on current basic research such as the fundamental research studies described in this article.

1.2 Spin and magnetic moment

While spin is naively interpreted as a micromagnet, its quantum mechanical properties are complicated and the spin sometimes behaves contrary to everyday experience. For example, one would associate the word *spin* with the spin of a tennis ball, where the magnitude of the ball's angular momentum can be varied continuously by applying a torque. However,

the *spin* of an electron cannot be varied continuously. To describe such peculiar properties of electron spin, we dare to provide a quantum mechanical explanation of electron spin in this article.

1.3 The Stern-Gerlach experiment

The fact that the magnitude of electron spin is not continuously variable but takes only discrete values was confirmed by the Stern-Gerlach experiment (**Fig. 1**) [1]. While tiny particles of silver were used in the actual experiment, the experimental results reflect the properties of electron spin. When electrons pass through space containing a magnetic field whose magnitude is graded in the perpendicular direction, their spins are decoupled into two states with different magnetic moments, as if they have been sorted (taking a bar magnet as an example, these are an up-spin state with the south (S) magnetic pole facing upward and a down-spin state with the S pole facing downward). A straightforward interpretation of the measured results leads to the conclusion that the magnetic moment of electron spins can take only two values: either the value corresponding to the upward direction or the value corresponding to the downward direction. Then, might there be electrons with a spin state that is neither upward nor downward vertically, such as those whose S pole is pointing horizontally among the electrons that have passed through the detector? To give the conclusion first: yes, there are

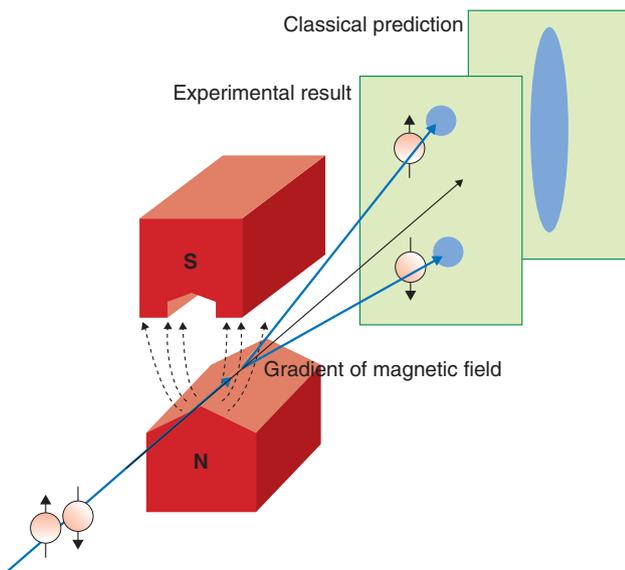


Fig. 1. Stern-Gerlach experiment.

such electrons. To understand why and to make correct predictions about spin-related phenomena in semiconductor devices, we need an accurate understanding of quantum mechanics.

1.4 Probability interpretation of the wave function and reduction of the wave packet

Two well-known concepts in quantum mechanics are probability interpretation (of the wave function) and reduction of the wave packet. Considering a one-dimensional world for simplicity, let us express the wave function of an electron by $\Psi(x)$.

Probability interpretation means the following. When we try to measure the location of an electron described by $\Psi(x)$, the probability that the electron is found at a position between x and $x+\Delta x$ is given by $|\Psi(x)|^2\Delta x$. Reduction of the wave packet is related to probability interpretation. It is a term that describes how an electron in a state with a wide probabilistic distribution $|\Psi(x)|^2$ (wave packet) before measurement collapses to a point at a certain position x as a consequence of the measurement.

1.5 Spin states other than up and down

Besides an electron's position, its spin state is also more correctly described by a wave function. If we apply the concept of reduction of the wave packet, then the Stern-Gerlach experiment is interpreted as the collapse of the wave function of electron spin to either an up state or a down state by measurement. We

note that the up/down direction is the direction of the magnetic field (z direction) within the experimental apparatus (detector).

Since we have freedom in the direction of the magnetic field to be set in the detector of the experimental apparatus, we can also select any arbitrary direction for the observation of a magnetic moment. We would like to emphasize that the phenomena we are explaining here are not related to the torque that the magnetic field in the detector may apply to the electron spin to change its orientation. We are saying that the orientation of the spin is varied as a result of the reduction of the wave packet.

To understand the above description, one needs to know that any state (wave function) of electron spin is described by a wave function $a|\uparrow\rangle_z + b|\downarrow\rangle_z = \begin{pmatrix} a \\ b \end{pmatrix}$,

which is a superposition of the up-spin $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv |\uparrow\rangle_z$ and

down-spin $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv |\downarrow\rangle_z$ wave functions. One needs to

accept the quantum mechanical conclusion that the probabilities of observing the up- and down-spin states upon measurement of spin state $a|\uparrow\rangle_z + b|\downarrow\rangle_z = \begin{pmatrix} a \\ b \end{pmatrix}$

are given by $|a|^2$ and $|b|^2$, respectively, where a and b are complex numbers such that $|a|^2 + |b|^2 = 1$. Moreover, if we perform the measurements on the same

spin state $a|\uparrow\rangle_z + b|\downarrow\rangle_z = \begin{pmatrix} a \\ b \end{pmatrix}$ while varying the direction

of the magnetic field in the detector, we will find a certain direction for which observation of only up-spin electrons will result (up-spin with respect to the chosen magnetic field direction). This direction is called the direction (or orientation) of the electron spin. Generally, electron spin orientations differ from one electron to another. However, the conclusion of quantum mechanics is that any electron spin can be expressed as $a|\uparrow\rangle_z + b|\downarrow\rangle_z$ by finding appropriate complex numbers a and b , whichever direction it is pointing in.

From the above discussion, we can conclude the following. If there are two spin states $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$ that are orientated in the $+x$ and $-x$ directions, respectively, we should be able to write them as a superposition of

$|\uparrow\rangle_z$ and $|\downarrow\rangle_z$, which ultimately leads to $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + |\downarrow\rangle_z)$ and $|\downarrow\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z - |\downarrow\rangle_z)$. If we interpret this

probabilistically, then the probability that a spin-up or spin-down state is detected (with respect to the z direction) is 50% upon the measurement of these

spins. Conversely, the state $a|\uparrow\rangle_z + b|\downarrow\rangle_z$ can also be written as $\frac{(a+b)}{\sqrt{2}}|\uparrow\rangle_x + \frac{(a-b)}{\sqrt{2}}|\downarrow\rangle_x$ using $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$. Therefore, if the direction of the magnetic field in the detector is in the x direction, the up- and down- spin states will be observed with probabilities $\frac{|a+b|^2}{2}$ and $\frac{|a-b|^2}{2}$, respectively.

We finally ask which direction is the electron spin $a|\uparrow\rangle_z + b|\downarrow\rangle_z$ pointing in? We provide only the conclusion here. The three-dimensional vector for the orientation of this spin is $(x, y, z) = (2\text{Re}(a^*b), 2\text{Im}(a^*b), |a|^2 - |b|^2)$, where $\text{Re}(a^*b)$ and $\text{Im}(a^*b)$ are the real and imaginary parts of the complex number a^*b , respectively.

1.6 Spin splitting due to the Rashba effect

In our research, we use a spin-orbit interaction called the Rashba effect [2] in controlling electron spins. A Hamiltonian^{*1} of the spin-orbit interaction generally takes the form $(\nabla V \times \mathbf{p}) \cdot \boldsymbol{\sigma}$. Using the confinement potential $V(z)$ of the quantum well for V , i.e., $\nabla V(z) = (0, 0, -eE_z)$, and $\hbar\mathbf{k}$ for \mathbf{p} (momentum), we obtain the Hamiltonian for the Rashba effect as $H_R = a_{so} \langle E_z \rangle (k_y \sigma_x - k_x \sigma_y)$. Here, a_{so} is a material-dependent intrinsic constant for the Rashba effect, $\langle E_z \rangle$ is the expectation value of E_z , and σ_x and σ_y are the Pauli spin matrices.^{*2} If we use a plane wave for the orbital part of the wave function for simplicity with a two-dimensional wave vector $\mathbf{k} = (k_x, k_y) = (k, 0)$, the wave functions (eigenfunctions) including the spin part are given by $\frac{e^{ikx}}{\sqrt{2}} \begin{pmatrix} i \\ \mp 1 \end{pmatrix}$ with respective eigenenergies $\frac{\hbar^2 k^2}{2m^*} \pm a_{so} \langle E_z \rangle k$. Thus, the energy splitting between the two states $\frac{e^{ikx}}{\sqrt{2}} \begin{pmatrix} i \\ \mp 1 \end{pmatrix}$ at a constant wave vector, which is caused by a spin-orbit interaction associated with the confinement potential of semiconductor heterostructures, is called the Rashba splitting^{*3} (Fig. 2).

*1 Hamiltonian: Operator that provides a physical quantity corresponding to energy in quantum physics.

*2 Pauli spin matrices: Three matrices $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ used to describe the spin angular momentum in quantum mechanics, and a vector form of them, i.e., $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.

*3 Rashba splitting: Spin splitting in the energy dispersion relation of an electron that is caused by the Rashba spin-orbit coupling effect.

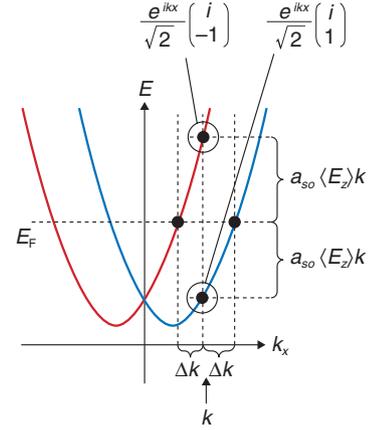


Fig. 2. Energy dispersion of two-dimensional electron system with the Rashba splitting.

1.7 Spin rotation

Consider a superpositioned wave function $\frac{e^{i(k-\Delta k)x}}{2} \begin{pmatrix} i \\ -1 \end{pmatrix} + \frac{e^{i(k+\Delta k)x}}{2} \begin{pmatrix} i \\ 1 \end{pmatrix} = ie^{ikx} \begin{pmatrix} \cos(\Delta kx) \\ \sin(\Delta kx) \end{pmatrix}$ that is composed of two wave functions associated with two points on the Fermi surface^{*4} $(k \pm \Delta k, 0)$, where $\Delta k = a_{so} \langle E_z \rangle m^* / \hbar^2$, in a system having the Rashba splitting. The spin part of this wave function at $x = 0$ is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, where the spin is orientated in the z direction. The electron spin states for $x \neq 0$ are $\begin{pmatrix} \cos(\Delta kx) \\ \sin(\Delta kx) \end{pmatrix} = \cos(\Delta kx) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin(\Delta kx) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the spin orientation of which is $(\sin(2\Delta kx), 0, \cos(2\Delta kx))$ in a three-dimensional vector. Thus, the spin rotates in the x - z plane as a function of the electron's position x . An electron in a semiconductor can be regarded as a wave packet having a central wave number. Since the propagation speed of a wave packet is given by its group velocity (Fermi velocity) $\frac{\hbar k}{m^*}$, an electron in uniform linear motion changes its position according to its Fermi velocity. Thus, the spin orientation will rotate accordingly. The angular velocity of the electron spin

*4 Fermi surface: Curved surface in the wave number space (k -space) defined by $E(\mathbf{k}) = E_F$, where $E(\mathbf{k})$ is the band energy dispersion of an electron in a solid and E_F is the Fermi energy. Regions in k -space that have energies lower than E_F are filled with electrons. The electron transport properties of a solid are usually closely related to the properties of the Fermi surface including its shape and how it interacts with phonons and impurities.

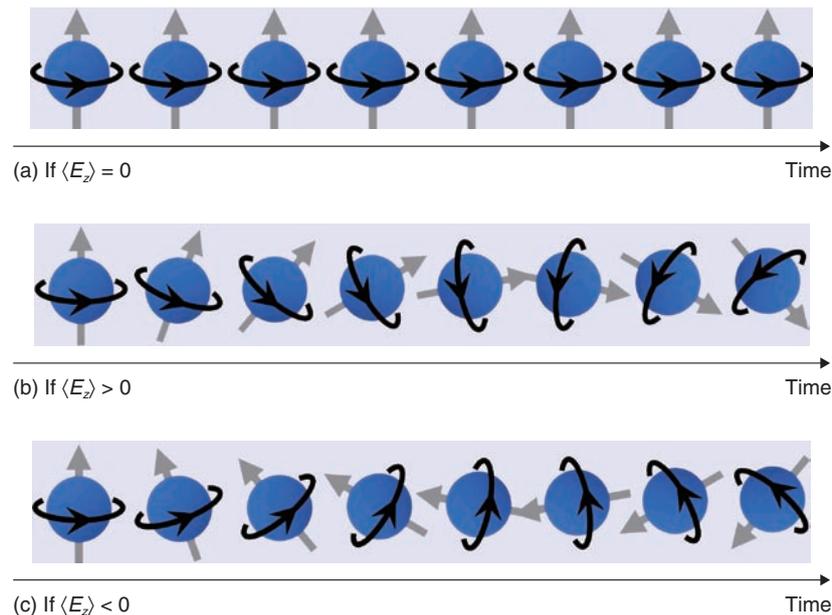


Fig. 3. Examples of spin precession.

rotation is given as $\omega = 2a_{so} \langle E_z \rangle k/\hbar$ in this case. The tunability of $\langle E_z \rangle$ by an external gate or a specific design of the heterostructure enables us to control the angular velocity of spin rotation, which is one of the greatest attractions of the Rashba effect. Some examples of spin rotation for an electron moving at the Fermi velocity are shown in **Fig. 3**, where a_{so} acts as a parameter indicating how easy it is to control the electron spin precession by an electric field $\langle E_z \rangle$.

In research on the Rashba effect up to now, it has been important to obtain a value called the Rashba coefficient α , which corresponds to $a_{so} \langle E_z \rangle$. Pieces of circumstantial evidence showing that the α values are controllable by a gate had been accumulated, though nobody had been able to show a quantitatively linear relationship between α and $\langle E_z \rangle$. The factors that have delayed the progress of this research include (1) a lack of experimental methods that enable one to measure the value of $\langle E_z \rangle$ within a quantum well directly and (2) uncertainty about the size of the Dresselhaus spin splitting (the spin splitting arising from the bulk inversion asymmetry), which coexists with the Rashba spin splitting. In collaborative work between Hokkaido University and NTT Basic Research Laboratories, we have demonstrated the linear relationship $\alpha = a_{so} \langle E_z \rangle$ quantitatively by using $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ semiconductor quantum wells (**Fig. 4**), which are lattice-matched to (001) InP; this led to the

successful quantitative determination of the value of a_{so} [3]. A key to our success was the coincidental discovery of an epitaxial wafer that has the property that the value of $\langle E_z \rangle$ can seemingly be varied from positive to negative values by means of gating. Careful experiments with this epitaxial wafer and a series of similar wafers resulted in our quantitative determination of a_{so} .

2. Latest results

2.1 Experimental procedure for determining a_{so}

The testpiece used in our experiment was a 10-nm-thick $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ semiconductor quantum well grown on the (001) plane of an InP substrate (with barrier layers of $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$). We controlled the electron density and the perpendicular electric field $\langle E_z \rangle$ within the quantum well by means of a surface gate in a field-effect-transistor structure (Fig. 4). We used a dilution refrigerator to cool the sample down to ~ 100 mK (electron temperature), in order to minimize the effect of thermal disturbance and increase the measurement sensitivity, and measured the electric resistance as a function of the perpendicular magnetic field (magneto-resistance) for various gate voltages. In these measurements, a distinctive phenomenon called the *weak antilocalization* appeared in the vicinity of zero magnetic field, owing to the

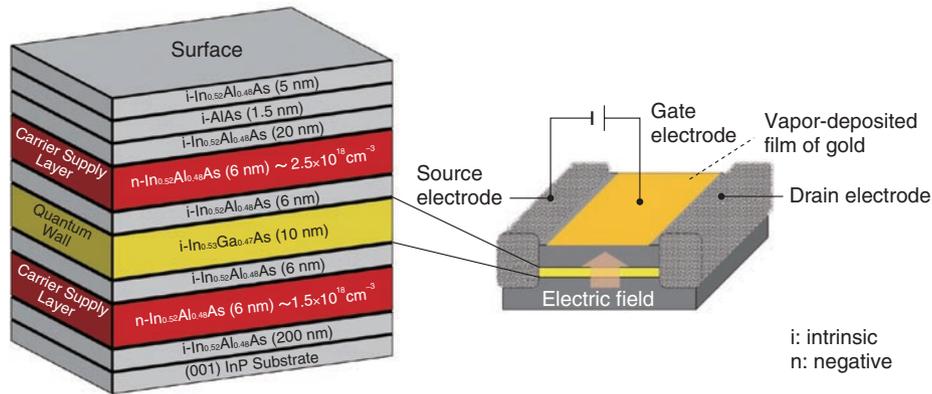


Fig. 4. Experimental sample and FET structure.

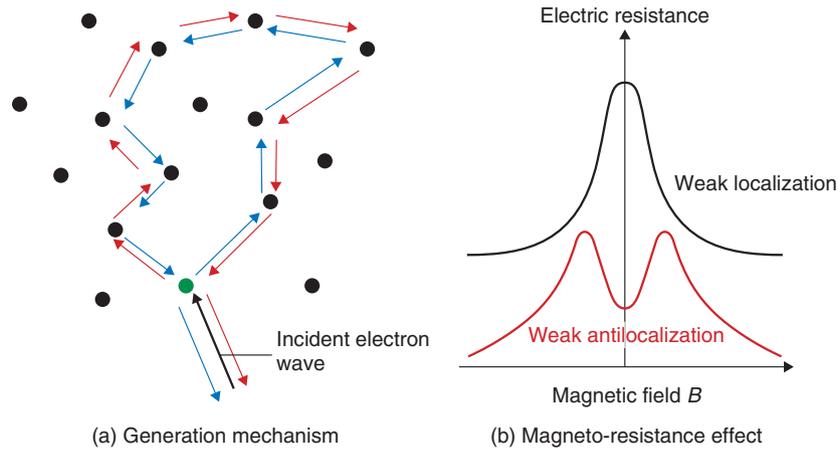


Fig. 5. Weak localization and weak antilocalization effects.

interference of the electron wave functions. We were able to determine the value of the Rashba coefficient α by theoretically analyzing this experimentally observed phenomenon [4].

2.2 Weak localization and weak antilocalization effects

Weak localization is a phenomenon that occurs when an electron is scattered sequentially in a loop by impurities distributed randomly in the conductor, whereby the electrons are weakly localized in the loop as a result of constructive interference of the wave functions. For example, if an electron happens to follow the closed path indicated by blue arrows in **Fig. 5(a)** after the injection of the electron as indicated by the black arrow, then the red path, which is

the time-reversed path of the blue one, should also be present quantum mechanically with the same probability as the blue path. Since the lengths of the two paths are exactly equal, constructive interference always occurs at the endpoint of the two paths (the position of the green impurity in **Fig. 5(a)**), if the electron propagation is not accompanied by spin rotation. This results in enhancement of the backscattering probability. This is the mechanism of weak localization.

The weak localization effect is partially destroyed if a magnetic field is applied perpendicular to the quantum well. As a result, a negative magneto-resistance appears in the vicinity of the zero magnetic field, as shown by the black curve in **Fig. 5(b)**. The *weak localization* effect observed in the vicinity of

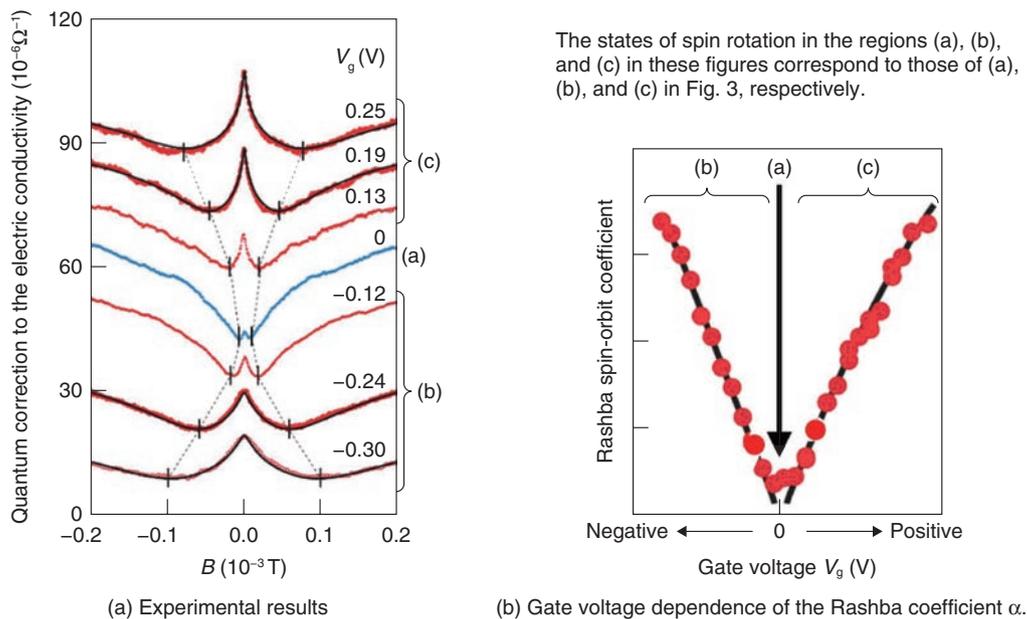


Fig. 6. Measurement of weak localization and weak antilocalization.

zero magnetic field is also suppressed by the rotation of spins, which is called *weak antilocalization*. In a more precise description, the mechanism of the *weak antilocalization* is related to the non-commutativity of spin rotation operators ($AB \neq BA$ for arbitrarily chosen A and B). This results in a positive magneto-resistance if the *weak antilocalization* is occurring (the red curve in Fig. 5(b)).

The results of our experiment are shown in **Fig. 6(a)**, where the electric conductivity of a quantum well sample is plotted (on the vertical axis) as a function of the magnetic field (horizontal axis) for various values of the gate voltage V_g . It is necessary to invert the + and - signs on the vertical axis (or invert up and down) when comparing the results with the schematic weak localization/antilocalization effects shown in Fig. 5(b) because the data plotted here are the electric conductances, not the resistances. Because V_g is a relative quantity, its value is set to 0 V when the Rashba splitting is minimized in this analysis. This experimental result shows that the weak antilocalization effect becomes stronger as the absolute value of V_g increases. Then, the value of the Rashba coefficient α can be extracted from the values of the fitting parameters that fit the experimental result best assuming a recently developed theoretical model. The gate voltage dependence of the Rashba spin-orbit interaction coefficient α of this sample

obtained from such analyses is shown in **Fig. 6(b)**. We note that the angular velocity of the electron spin rotation rises faster as α increases. The fact that we successfully made accurate predictions of the values of α as a function of the gate voltage V_g as in Fig. 6(b) indirectly means that we succeeded in controlling the angular velocity of the electron spin rotation freely by means of the gate of a transistor. For example, we can rotate the spins of electrons within a semiconductor about some specific axis ((b) in Fig. 3), halt the rotation ((a) in Fig. 3), and rotate them in reverse ((c) in Fig. 3) all by means of a gate. As a result, we expect that the knowledge acquired in this research will bring us one step closer to being able to utilize electron spins in future electronic devices such as quantum computers and ultra-low-power logic devices.

References

- [1] W. Gerlach and O. Stern, "Das Magnetische Moment des Silberatoms," *Zeitschrift für Physik*, Vol. 9, No. 1, pp. 353–355, 1922.
- [2] Yu. A. Bychkov and E. I. Rashba, "Oscillatory Effects and the Magnetic Susceptibility of Carriers in Inversion Layers," *J. Phys. C: Solid State Phys.*, Vol. 17, No. 33, pp. 6039–6045, 1984.
- [3] S. Faniel, T. Matsuura, S. Mineshige, Y. Sekine, and T. Koga, "Determination of Spin-Orbit Coefficients in Semiconductor Quantum Wells," *Phys. Rev. B*, Vol. 83, No. 11, 115309, 2011.
- [4] T. Koga, J. Nitta, T. Akazaki, and H. Takayanagi, "Rashba Spin-Orbit Coupling Probed by the Weak Antilocalization Analysis in InAlAs/InGaAs/InAlAs Quantum Wells as a Function of Quantum Well Asymmetry," *Phys. Rev. Lett.*, Vol. 89, No. 4, 046801, 2002.



Takaaki Koga

Associate Professor, Graduate School of Information Science and Technology, Hokkaido University.

He received the B.S. degree in engineering from the University of Tokyo in 1992 and the M.S. and Ph.D. degrees in applied physics from Harvard University, USA, in 1995 and 2000, respectively. He joined NTT Basic Research Laboratories as a research associate in 2000 and studied semiconductor spintronics. In 2004, he moved to Hokkaido University as an associate professor. He is currently engaged in an experimental study of spin-orbit interaction in semiconductors. He is a member of the American Physical Society, the Physical Society of Japan (JPS), and the Japan Society of Applied Physics (JSPA).



Yoshiaki Sekine

Researcher, Spintronics Research Group, NTT Basic Research Laboratories.

He received the B.S. degree in engineering from Hokkaido University in 1994 and the M.S. and Ph.D. degrees in science from the University of Tokyo in 1996 and 1999, respectively. He joined NTT Basic Research Laboratories in 1999 and has been studying semiconductor spintronics. He is currently engaged in an experimental study of spin-orbit interaction in semiconductors. He is a member of JPS and JSPA.
